Advanced Mechanics

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5 problems (total of 50 points).
The solution of every problem on a separate piece of paper with name and student number.
Use the attached formula list where necessary.

Problem 1 (7 pnts in total)

A damped oscillator $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F(t)/m$ with $\omega_0^2 = 2\beta^2$ is driven by a force that is exponential in time,

$$F(t) = F_0 e^{-t/\tau} \text{ for } t > 0$$

with $\beta = 1/\tau$ and $F(t) = 0$ for $t < 0$.

3 pnts a. Give the expression for $x(t)$ using the Greens function.

4 pnts b. Solve for $x(t)$.

Problem 2 (14 pnts in total)

Calculate the shortest path on a conical surface, given by $z = 1 - r$ in cylindrical co-ordinates. r is the distance in the (x, y) plane and ϕ is the angle in this plane wit the x-axis.

3 pnts a. Give the form for the expression to minimize.

2 pnts b. Give the resulting Euler equations.

4 pnts c. Show that the general solution can be written as

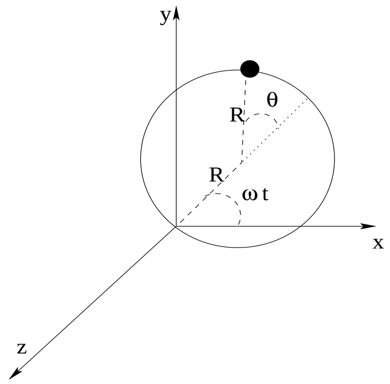
$$\frac{r_0}{r} = \cos\left(\frac{\phi - \phi_0}{\sqrt{2}}\right)$$

2 pnts d. Give r_0 and ϕ_0 for the path from $(z, r, \phi) = (0, 1, 0)$ to $(0, 1, \pi)$.

3 pnts e. Calculate the pathlength going from $(z, r, \phi) = (0, 1, 0)$ to $(0, 1, \pi)$.

Problem 3 (10 pnts in total)

A bead slides without friction on a hoop that rotates with constant angular velocity ω about an axis perpendicular to the plane of the hoop and passing through the edge of the hoop (see the figure). The angle θ, measures the displacement of the bead. Note that this problem ignores both friction and gravity.



3 pnts a. Write the Lagrangian in generalized coordinates.

3 pnts b. Determine the equation of motion of the bead.

2 pnts c. Determine the frequency of small oscillations.

2 pnts d. Determine the generalized momentum, p_θ.

Problem 4 (8 pnts in total)

Given a particle with mass m and angular momentum in the $\hat{z}$ direction of magnitude l that moves in a central force field given by

$$F(r) = -\frac{k_2}{r^2} - \frac{k_4}{r^4} .$$

The solution will be written as $r(t) = \rho + \epsilon x(t)$ where ρ and ϵ are constant, time independent

2 pnts a. Give the equation of motion.

3 pnts b. Find ρ for the case that $\epsilon = 0$.

3 pnts c. Find the frequency of small oscillations around $r = \rho$.

Problem 5 (11 pnts in total)

A spring with a mass m attached hangs from the ceiling. To this mass another identical spring with attached mass is coupled. Both masses are equal. Both springs are weightless with spring constant k and length l when not stretched. We will consider only a motion in the vertical direction.

2 pnts a. Write down the Lagrangian for this problem.

3 pnts b. Give the equations of motion for both masses.

3 pnts c. Give the equilibrium positions of the masses.

3 pnts d. Give the frequencies of the eigen modes for this set of coupled oscillators.